SEMINAR REPORT

ON

“CONTROL OF A GROUP OF MOBILE ROBOTS”

SUBMITTED BY

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(Reg. No: 13121409)

in partial fulfilment of the award of the Degree

of

Bachelor of Technology

in

ELECTRONICS & COMMUNICATION ENGINEERING

of

COCHIN UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING (DEPT. OF ECE)

COLLEGE OF ENGINEERING, KIDANGOOR

(Under Co-operative Academy of Professional Education (CAPE), Estd. by the Govt. of Kerala)

KIDANGOOR SOUTH P.O, KOTTAYAM - 686583

NOVEMBER 2014

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CERTIFICATE

Date: …………………

*Certified that this seminar work titled "**CONTROL OF A GROUP OF MOBILE ROBOTS**" is the bonafide record of the work done by **Mr. EMIL ZACHARIA GEORGE (Reg. No: 13121409)** of **B.Tech. Seventh Semester, Electronics And Communication Engineering**, towards the partial fulfilment of the requirement for the award of the Degree of Bachelor of Technology, by the Cochin University of Science and Technology **(CUSAT)** , in the year 2014.*

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ACKNOWLEDGEMENT

Any challenging work can be satisfactorily completed only with the support and guidance of learned people. I owe to many great people who constantly supported and motivated me to come up with this seminar.

First and foremost, I am grateful to ***God almighty***, for his divine grace bestowed on me, during the selection of this topic and following studies related to it. I am extremely thankful to ***Dr. Roobin V Varghese***, the respected Principal of College of Engineering, Kidangoor. I am also thankful to ***Mrs. Deepthy Mathew***, Head of Department of Electronics and Communication Engineering, College of Engg., Kidangoor, for her co-operation and guidance with regard to presentation of seminar & preparation of this report.

I thank wholeheartedly my seminar co-ordinators ***Ms. Arya Rajan*** & ***Ms. Arathy S***, Asst. Professors in ECE Dept., College of Engg. Kidangoor, who acted as mentors, providing me their valuable suggestions and corrections. I extend my sincere thanks to ***all faculty members*** of ECE Department, especially ***Mrs. Shiney Thankachan & Mrs. Jismi Babu***, Asst.Professors for their help and support.

Last but not least, I express my gratitude to ***my parents*** and ***my friends*** for their co-operation and advice which helped in the completion of this seminar.

ABSTRACT

The paper proposes a new method of controlling a group of mobile robots based on formation abstraction. The shape of a formation is represented by a deformable polygon, which is constructed by bending a rectangle, to go through narrow spaces without colliding with obstacles. If the trajectory of the front end point, as well as the width and the length of the formation are given, the formation automatically reshapes itself to fit the area through which the front part of the group has already safely passed. Furthermore, the robots continuously try to optimize their positions to decrease the risk of collisions by integrating a decentralized locational optimization algorithm into the formation control.

The study shows that the objective function, taking into account the distance between robots, does not decrease for fixed and non-convex polygonal formation shapes if the zero-order hold control is applied for a sufficiently short sampling period. The influence of the decentralized locational optimization algorithm on the objective function in the case of variable formations is also analyzed.

The effectiveness of the proposed method is demonstrated in both simulations and real robot experiments.

CONTENTS

LIST OF FIGURES

CHAPTER 1:
INTRODUCTION

A mobile robot is an automatic machine that is capable of locomotion. In the case of a group of mobile robots, it is supposed that a desired collective behavior emerges from the interactions between the robots and interactions of robots with the environment. Coordination of multiple mobile robots has been a significant field of research for possible applications such as exploration, surveillance, mapping of unknown environments, and the transport of large objects. This paper focuses on the fundamental problem of moving a number of robots as a whole to a target area. There are several types of approaches to deal with this problem.

The approach most closely related to the one we use is based on formation abstraction. The whole group of robots is represented by a simple geometric shape, such as a rectangle or an ellipsoid. The control method aims to keep a group of point robots inside a given geometric shape, which is dynamically changing. Thus, some robots might become significantly closer to each other as the formation reshapes, even if the area of the shape is maintained. More recent studies on formation abstraction introduce a velocity constraint or an additional feedback law for each robot to move away from other robots inside a prescribed safety region. For such decentralized feedback laws based on local information, it is inevitably difficult to guarantee that the requirements of both collision avoidance and formation shape control are satisfied, since these requirements often conflict with each other.

However, more flexibility of the formation is required to move in complex environments. One way for a large group to go through a narrow space without colliding with obstacles is to form a long slender shape and bend the formation to fit the area through which the front part of the group has already safely passed. As a result, if the front part of the group does not collide with obstacles, the following parts can pass through the same area without collision. Similar strategies have been adopted in the operation of snakelike robots, which are typically composed of three or more segments connected serially

Here, we use a new method of controlling a group of mobile robots based on formation abstraction. In rectangular formations, we use a polygonal formation that can bend to go through narrow spaces without colliding with obstacles. A polygonal formation is constructed based on a serial link in the middle of the formation. The polygon is bent by changing the joint angles of the serial link Therefore, if the trajectory of the front end point as well as the width and the length of the formation are given, the formation automatically reshapes to fit the area through which the front part of the formation has already safely passed. Another feature of this method is that actions to decrease the risk of collisions are started before other robots enter the safety region prescribed for collision avoidance.

Thus, we consider the objective function which is defined as the minimum of the distances between robots and the distances from robots to the boundary of the polygonal region, in order to quantify the risk of violation of the requirements of collision avoidance and formation shape control. In the proposed method, all the robots continuously try to achieve the optimal disposition in which the objective function is maximized. Due to the non-smooth and decentralized properties of the control law, it is not straightforward to guarantee the optimality of the robot positions even in cases where the formation shape is fixed and convex. Thus, we show that the objective function does not decrease for fixed and non-convex polygonal formation shapes if the zero-order hold control is applied for a sufficiently short sampling period. In the variable formation case, the cost function is not necessarily non-decreasing. The effectiveness of the proposed method is demonstrated in both simulations and real robot experiments.

CHAPTER 2:

CONTROL OBJECTIVE

Consider n robots with the following kinematic model:

$$\dot{q}_i = u_i, \quad i = 1, \ldots, n$$

where $q_i \in \mathbb{R}^2$ and u_i are the absolute position and the control input of the robot i, respectively. The set of robots closest to robot $i \in \{1, \ldots, n\}$ is defined as $\mathcal{N}_i := \left\{ j_i \,\middle|\, \|q_i - q_{j_i}\| = \min_{j \neq i} \|q_i - q_j\| \right\}$.

It is assumed that the robot $i \in \{1, \ldots, n\}$ knows its own position q_i and the relative position $q_{j_i} - q_i$ of each nearest robot $j_i \in \mathcal{N}_i$. This implies that there is always at least one robot in the sensing region of every robot.

2.1) FORMATION ABSTRACTION

By formation abstraction, we mean that the gross position and orientation as well as the shape of a group of robots are described by a smaller number of states, which are called abstract states, than the dimension 2n of $Q := [q_1^T, \ldots, q_n^T]^T$. The group is represented by a simple geometric shape, inside of which each robot is controlled to maintain. We aim to achieve formation abstraction exhibiting two properties:

1) Only a few abstract states independent of n need to be chosen according to the information on the environments.

2) The rest of the abstract states are automatically determined by a prescribed algorithm according to the few abstract states given before.

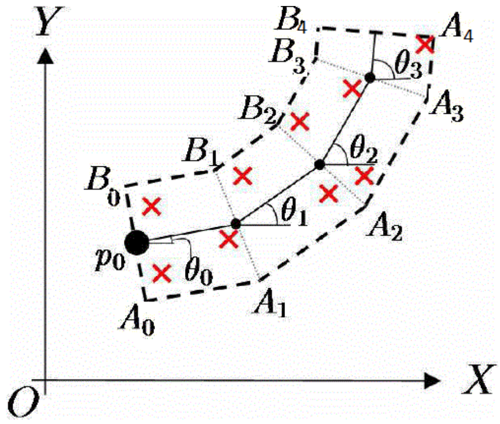

Figure 2.1: Polygonal region *P*.

We consider a formation shape described by the polygon *P* as shown by the dashed line. To construct the polygon *P*, we first consider a serial-link structure as shown in the middle of the polygon. Let σ_1 be the sum of the lengths of all the links, and the length of each link except for the last link is assumed to be given as a constant L. Then, the number of links is defined as m:= [σ1/L], and the length of the last link is $[\sigma_1-(m-1)L]$. The front end of the serial-link structure is located at p_0, and the orientation of each link is defined as θ_k (k = 0, 1, ..., m−1). Two of the edges of the polygon in are orthogonal to the first and the last links, respectively. The rest of the edges are parallel to one of the links and have a distance $\sigma_2/2$ from the link. Such relationships between the polygon and the serial link are always maintained, when the formation reshapes.

Two ways to change the shape of the formation are:

1) By changing the relative angles between adjacent links, i.e., $\theta_k-\theta_{k-1}$ (k=1,.., m). As a special case, where $\theta_0= \theta_1=\ldots= \theta_{m-1}$, the formation shape is a rectangle, which has length σ_1 and width σ_2.
2) By changing σ_1 and σ_2. When the group goes into a narrow space, the width σ_2 should be decreased, while the length σ_1 is accordingly increased to maintain the area of the polygon so that it can accommodate all the robots.

The target value of $\sigma: = [\sigma_1, \sigma_2]^T$ is defined as $\sigma_d= [\sigma_{d1}, \sigma_{d2}]^T$. If σ_d given, then σ can be easily determined such that σ converges to σ_d, for example, by

$\dot{\sigma}_1 = K_1(\sigma_{d1} - \sigma_1)$ and $\dot{\sigma}_2 = K_2(\sigma_{d2} - \sigma_2)$ where K_1 and K_2 are given positive constants.

We assume that p_0, θ_0, and σ_d are given by a human operator who has information on the environments. One way for the formation to go through a narrow space without colliding with obstacles is to control θ_k (k = 1, .. , m−1) such that each link in the middle of the formation tracks the path of the preceding link. The first link is not allowed to move backwards or sideways. More precisely, the trajectories of p_0 and θ_0 are determined such that

$$\dot{p}_0 = -\begin{bmatrix} v_0 \cos\theta_0 \\ v_0 \sin\theta_0 \end{bmatrix}, \quad \dot{\theta}_0 = \omega_0$$

where p0 is allowed to move forward only, i.e., v0 > 0. In addition, the joint angles are limited as follows,

$$|\theta_k - \theta_{k-1}| \leq \pi/2, \quad k = 1, \ldots, m-1.$$

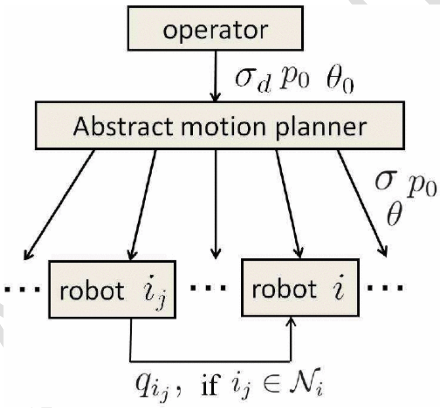

Figure 2.2: Information flow.

We assume that $\theta := [\theta_0, \ldots, \theta_{m-1}]^T$ and σ are determined from p_0, θ_0, and σ_d by the abstract motion planner on a central computer and that p_0 , θ and σ are broadcasted to each robot.

To derive a control law that maintains all the robots in *P*, we introduce the coordinate transformation $T(p_0, \theta)$ from q to $z = [r, l]^T$ in the r–l frame as shown below.

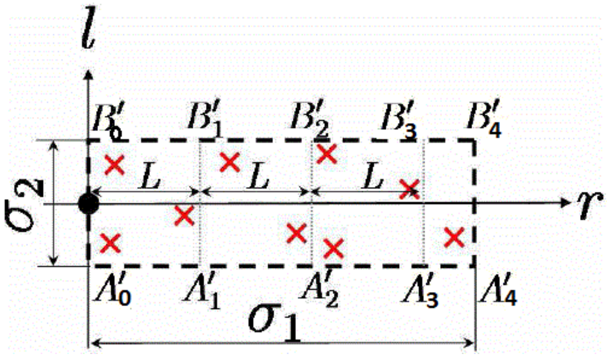

Figure 2.3: Rectangular region R.

The subscript "(p_0, θ)" indicates that z = T(p_0, θ) (q) depends on p_0 and θ, as well as on q. Roughly speaking, l in Fig. 2.3 represents the deviation from the closest link, while r represents the deviation from p_0 measured along the links. The points (A1, A2, B2, B1) in Fig. 2.1 are mapped to the points (A1', A2', B2', B1') in Fig. 2.3. In order to simplify the transformation, we assume that the joint angle and σ_2 satisfy the following assumption:

$$|\theta_k - \theta_{k-1}| \leq 2\tan^{-1}\frac{L}{\sigma_2}, \quad k = 1, \ldots, m-1.$$

Under this assumption, the polygon P is always divided into m trapezoids. We define the rectangular region in Fig. 2.3 as

$$\mathcal{R} := \{[r, l]^T \in \mathbb{R}^2 \mid 0 < r \leq \sigma_1, |\, l \,| < \sigma_2/2\}.$$

Then, q ∈ *P* if and only if z ∈ R. One of our control objectives, which we refer to as the main task, is to keep all the robots in the polygon *P*, while σ, p_0, and θ are changed. It is assumed that σ_1 and σ_2 are chosen such that $q_i \in P$ for any robot i ∈ {1,.., n} at the initial time. We also assume that $q_i(0) \neq q_j(0)$ for any j ∈{1, …, n} \ {i}.

2.2) LOCATIONAL OPTIMIZATION

Another control objective, which we call the subtask, is to decrease the risk of collisions by keeping as long distance as possible between robots as well as between a robot and the boundary of *P*. Consider the variable V:= [Q^T, σ^T, θ^T, p_0^T]^T. Consider $\mathcal{M}_i := \partial\mathcal{P} \cup \bigcup_{j_i \in \mathcal{N}_i} \left\{\frac{1}{2}(q_i + q_{j_i})\right\}$ in which ∂P generally depends on (σ, θ, p_0).

To represent the minimum distance from the robot i∈{1,.., n} to other robots or the boundary of *P*, we define $H_i(V) := \min_{q \in \mathcal{M}_i} \|q_i - q\|, \quad i = 1, \ldots, n.$

Therefore $H_i(V)$ is a function of σ, θ, p_0 & Q. Note that half of the distance between two robots is considered in $H_i(V)$, since twice the distance is needed to prevent the collision with other robots compared with the collision with the boundary of *P*. This is because a robot needs to take into account the size of another robot as well as its own size in order to avoid colliding with other robots. The objective function for the whole group: $\mathcal{H}(V) := \min_{i \in \{1,\ldots,n\}} H_i(V).$

We introduce a control law which aims to increase the value of $\mathcal{H}$(V) as much as possible for a given (σ, θ, p_0) by controlling Q. In the case of point robots, each robot does not collide with other robots and the boundary of *P*, if $\mathcal{H}$(V) is not decreasing, because of the assumption that $q_i(0) \in P$ and $qi(0) \neq q_j(0)$ for $i \in \{1, \ldots, n\}$ & $j \in \{1, \ldots, n\} \setminus \{i\}$.

CHAPTER 3:
CONTROL METHOD

The control input for each robot can be divided into two components, as follows:

$$u_i = u_{mi} + u_{si}, \quad i = 1, \ldots, n$$

where u_{mi} and u_{si} are the control inputs for the main task and the subtask, respectively.

3.1) CONTROL LAW FOR THE MAIN TASK

The control input u_{mi} for the main task is given such that $z_i = T(p_0, \theta)(q_i)$ satisfies $\dot{r}_i = \frac{r_i}{\sigma_1}\dot{\sigma}_1, \quad \dot{l}_i = \frac{l_i}{\sigma_2}\dot{\sigma}_2, \quad i = 1, \ldots, n$ when $u_i = u_{mi}$.

Proposition 1: For the above system, we have $0 < r_i(t) < \sigma_1(t)$, $| l_i(t) | < \sigma_2(t)/2$ $\forall$ $t > 0$, if it is satisfied at $t = 0$. In the same way, if $z_i(0) \neq z_j(0)$ for another robot $j \in \{1,\ldots, n\} \setminus \{i\}$, it can be proved that $z_i(t) \neq z_j(t)$ for any $t > 0$. This implies that $q_i(t) \neq q_j(t)$ for any $t > 0$ if $q_i(0) \neq q_j(0)$, since $T(p_0,\theta)$ is a one-to-one transformation from q to z. However, unlike point robots, real robots collide when their positions are too close, even if $z_i(t) \neq z_j(t)$.

Consider a rectangular formation changing the width and length as $\dot{\sigma}_1 > 0$ and $\dot{\sigma}_2 < 0$, respectively. In this case, the closed-loop system for the robot i without u_{si} is described as $\dot{X}_i = \frac{X_i}{\sigma_1}\dot{\sigma}_1, \ \dot{Y}_i = \frac{Y_i}{\sigma_2}\dot{\sigma}_2, \ i = 1, \ldots, n$

where (X_i, Y_i) is the XY coordinate of q_i. Thus, the relative position of the robot i with another robot $j \in \{1, \ldots, n\} \setminus \{i\}$ is changed as follows:

$\dot{X}_i - \dot{X}_j = \frac{X_i - X_j}{\sigma_1}\dot{\sigma}_1, \quad \dot{Y}_i - \dot{Y}_j = \frac{Y_i - Y_j}{\sigma_2}\dot{\sigma}_2.$ The distance between two robots i and j in the Y -direction, $|Y_j - Y_i|$, is obviously decreased due to $\dot{\sigma}_2 < 0$. This implies that the two robots, whose X-coordinates are close, i.e., $X_i \approx X_j$, could collide, even if the Y -coordinates of the robots I and j are initially large enough to avoid collision. In order to avoid both collisions and achieve the target formation shape in this case, the distance in the X-direction $|X_i - X_j|$ has to be increased. However, since $X_i \approx X_j$, almost no change is made to $|X_i - X_j|$. In other words, although an additional space is generated in the X-direction due to $\dot{\sigma}_1 > 0$, the two robots, which are close to each other in the X-direction, do not use the additional space and remain close to each other.

To overcome the limitation of $u_i = u_{mi}$, we next introduce an additional control law that always tries to optimize the positions of the robots in formation.

3.2) CONTROL LAW FOR THE SUBTASK

We generalize the control method for convex regions for the locational optimization in the non-convex region as follows:

$$u_{si} = \hat{K} \cdot \mathrm{Ln}(S_i), \quad i = 1, \ldots, n$$

where $S_i := \mathrm{co}\left\{ \frac{q_i - q^*}{\|q_i - q^*\|} \,\middle|\, q^* \in \arg\min_{q \in \mathcal{M}_i} \|q_i - q\| \right\}$ & $\hat{K}$ is a positive constant. Here, co(S) denotes the convex hull of a set $S \subset R^2$, and Ln(S) is the least-norm element of the closure $\bar{S}$ of S. the control law aims to move the robot i away from the convex hull of the closest points in $\mathcal{M}_i$.

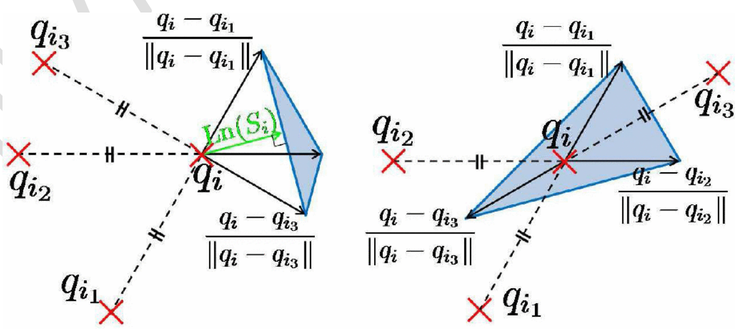

Figure 3.1: Schematic of Ln(S_i).

Figure 3.1 illustrates Ln(S_i) in the case where three points $\{q_{i1}, q_{i2}, q_{i3}\}$ in $\mathcal{M}_i$ are closest to q_i. Figure 3.1(left) illustrates a case where S_i does not contain 0_2, while Figure 3.1(right) illustrates a case where Si contains 0_2.

Let Ed(P) := {e_1,..., e_M } denote the set of edges of P when the number of edges is M. We denote by Cp_e (q_i) the closest point from q_i to the edge e ∈ Ed(P). If three regions D_L := {q ∈ $\mathbb{R}^2$ | (q − v_L) · (v_R − v_L) ≤ 0, (q − v_R) · (v_L − v_R) ≥ 0}, D_M := {q ∈ $\mathbb{R}^2$ | (q − v_L) · (v_R − v_L) ≥ 0, (q − v_R) · (vL − v_R) ≥ 0}, and D_R := {q ∈ R2 | (q − v_L) · (v_R − v_L) ≥ 0, (q−v_R) · (v_L−v_R)≤0} are defined using the coordinates of the vertices v_L and v_R of e, Cp_e (q_i) can be written as

$$\mathrm{Cp}_e(q_i) = \begin{cases} v_L, & \text{if } q_i \in D_L \\ v_L + \dfrac{(q_i - v_L)\cdot(v_R - v_L)}{\|v_R - v_L\|^2}(v_R - v_L), & \text{if } q_i \in D_M \\ v_R, & \text{if } q_i \in D_R. \end{cases}$$

By using Cp_e (q_i) above, H_i is rewritten as

$$H_i(V) = \min\left\{\min_{j_i \in \mathcal{N}_i} F_{(i,j_i)}(V), \min_{e \in \mathrm{Ed}(\mathcal{P})} G_{(i,e)}(V)\right\}$$

where $F_{(i,j)}(V) := \frac{1}{2}\|q_i - q_j\|, G_{(i,e)}(V) := \|q_i - \mathrm{Cp}_e(q_i)\|.$

Thus,

$$S_i = \mathrm{co}\left(\left\{\frac{q_i - q_{j_i}}{\|q_i - q_{j_i}\|}\,\middle|\, F_{(i,j_i)} = H_i, j_i \in \mathcal{N}_i\right\} \cup \left\{\frac{q_i - \mathrm{Cp}_e(q_i)}{\|q_i - \mathrm{Cp}_e(q_i)\|}\,\middle|\, G_{(i,e)} = H_i, e \in \mathrm{Ed}(\mathcal{P})\right\}\right)$$

Therefore, u_{si} can be computed using the information on the nearest robot of the robot i and on a finite number of the edges of P.

CHAPTER 4:
ANALYSIS OF THE OBJECTIVE FUNCTION

4.1) GENERALIZED GRADIENT OF THE OBJECTIVE FUNCTION

The objective function $\mathcal{H}$(V) is not differentiable in general, since the robot having the minimum value of H_i (V) could be switched. Further, H_i(V) is not differentiable, since the closest point q∗∈ M_i to the robot i could be switched discontinuously. Therefore, the theoretical analysis is based on the generalized gradient of $\mathcal{H}$(V).

In order to derive the generalized gradient of $\mathcal{H}$ (V), we first rewrite H_i in as $H_i(V) = \min\left\{\min_{j\in\{1,\ldots,n\}\setminus\{i\}} F_{(i,j)}, \min_{e\in \mathrm{Ed}(\mathcal{P})} G_{(i,e)}\right\}$ without using Ni.

Proposition 2: If $q_i \neq q_j$ and $q_i \in$ int(P), then the functions $F_{(i,j)}$(V) and $G_{(i,e)}$(V) are continuously differentiable for each (i, j) ∈ IJ and (i, e) ∈ IE, where

IJ := {(i, j) | i ∈ {1, . . . , n}, j ∈ {1, . . . , n}, i ≠j}

IE := {(i, e) | i ∈ {1, . . . , n}, e ∈ Ed(*P*)} and int(S) denotes the interior of a set S.

Since Proposition 2 implies that $-F_{(i,j)}$ and $-G_{(i,e)}$ are continuously differentiable, they are locally Lipschitz and regular.

Thus, we obtain
$$\begin{aligned}\partial H_i(V) = &\ \mathrm{co}(\{\nabla F_{(i,j)}(V) \mid F_{(i,j)} = H_i\}\\ &\cup \{\nabla G_{(i,e)}(V) \mid G_{(i,e)} = H_i\})\end{aligned}$$

It is seen that $\mathcal{H}$ (V) is regular and that $\partial\mathcal{H}(V) = \mathrm{co}\{\partial H_i(V) \mid i \in I_H(V)\}$.

Finally, the generalized gradient of $\partial\mathcal{H}$ is written as
$$\begin{aligned}\partial\mathcal{H}(V) = &\ \mathrm{co}(\{\nabla F_{(i,j)}(V) \mid (i,j) \in I_F(V)\}\\ &\cup \{\nabla G_{(i,e)}(V) \mid (i,e) \in I_G(V)\})\end{aligned}$$

where
$$I_F(V) := \{(i,j) \in \mathcal{IJ} \mid F_{(i,j)}(V) = \mathcal{H}(V)\}$$
$$I_G(V) := \{(i,e) \in \mathcal{IE} \mid G_{(i,e)}(V) = \mathcal{H}(V)\}.$$

4.2) CASE OF FIXED & NON-CONVEX FORMATION

Here we consider the case where the polygonal region *P* is fixed. In such cases, we have $u_{mi} = 0_2$ (i = 1,..., n), since $\dot{\sigma} = 0_2$, $\dot{\theta} = 0_m$, and $\dot{p}_0 = 0_2$. Thus, the overall system of n robots is described in a vector form as

$\dot{Q} = U_s, \quad U_s := [u_{s1}^T, \ldots, u_{sn}^T]^T$ which is equivalently

$\dot{V} = \mathcal{U}_s, \quad \mathcal{U}_s := [U_s^T, 0_{m+4}^T]^T$.

Due to the non-smoothness of u_s, it is not straightforward to s$\mathcal{H}$w that is not decreasing, even in cases where *P* is fixed. One possible way to show this is to apply the non-smooth Lyapunov theory based on the Filipov solution to non-smooth dynamical systems by regarding $\mathcal{H}$ as a non-smooth Lyapunov function. However, it is difficult to show that each element of the set-valued Lie derivative of $\mathcal{H}$ along the Filipov solution is non-negative even in cases where *P* is a convex region, since the control law is decentralized in the sense that only the information on the closest point q^* from each robot is used. This topic is discussed based on the property that each element in the set-valued Lie derivative of $\mathcal{H}$ with respect to U_s is equivalent to $\zeta \cdot U_s$ for any $\zeta \in \partial H$.

However, it is uncertain whether this property holds except for the case where the set $\mathrm{argmin}_{q \in M_i} \|q_i - q^*\|$ of the closest point q^* is a singleton. Thus, unlike these existing studies, we show that $\mathcal{H}$(V) is not decreasing if the zero-order hold control is applied for a sufficiently short sampling period Δt. It is reasonable to focus on the zero-order hold control, since it is used in many practical cases.

4.3) CASE OF VARIABLE FORMATION

In cases where (σ, θ,$\mathcal{H}_0$) is variable, the cost function $\mathcal{H}$(V) is not necessarily non-decreasing, even if the locational optimization algorithm is used. Therefore, we aim to guarantee that the input for the subtask u_{si} (i = 1,…, n) makes the value of $\mathcal{H}$(V) greater than that in the case without u_{si}. If this is guaranteed, the disposition of the robots is improved in terms of both the collision avoidance and the formation shape control, since $\mathcal{H}$(V) takes into account the distance between a robot and a boundary of the polygon, as well as the distance between robots. The property $\mathcal{H}$(V) > 0, which is achieved without u_{si} (i = 1,.., n) is also achieved when u_{si} is applied in addition to u_{mi}. Thus, in particular, $q_i \in P$ (i = 1,..., n) is also achieved at t >0, if it is initially achieved at t = 0.

The overall system of n robots is described as

$$\dot{V} = \mathcal{U} = \mathcal{U}_m + \mathcal{U}_s$$
$$\mathcal{U} := [u_1^T, \ldots, u_n^T, \dot{\sigma}^T, \dot{\theta}^T, \dot{p}_0^T]^T$$
$$\mathcal{U}_m := [u_{m1}^T, \ldots, u_{mn}^T, \dot{\sigma}^T, \dot{\theta}^T, \dot{p}_0^T]^T .$$

However, the control input u_{si} (i = 1,..., n) for the subtask rarely becomes 0 in the transient state of (σ, θ, p_0) where $u_{mi} \neq 0$, since the distance between robots or the distances between a robot and a boundary of *P* are dynamically changing. On the other hand, in the steady state of (σ, θ, p_0) where $u_{mi} = 0$, it is proved that $\mathcal{H}$(V) is not increased by u_{si}.

CHAPTER 5:
SIMULATIONS

5.1) CASE OF VARIABLE FORMATION

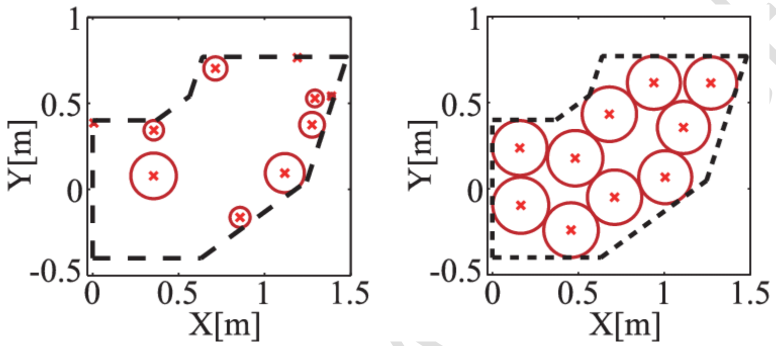

Figure 5.1: Initial (left) and final (right) robot positions in Section 5.1.

For a non-convex polygon as shown in the dashed line in Figure 5.1, the control input us for the subtask is applied using the zero order hold with sampling time T_s = 0.1 and 0.01 s. The gain of the control law in (14) is ˆK = 0.1. We randomly generate 100 sets of initial positions of ten robots, and test whether the objective function H is non-decreasing or not. To this end, we compute the difference in H for each sampling interval and focus on the worst case in the sense that the largest decrease in H for a sampling interval T_s = 0.1 is observed. The initial positions of the robots in this case are shown by "×" in Figure 5.1 (left), while the terminal positions in the case of T_s = 0.1 are shown in Figure 5.1 (right). Note that the radius of the circle around each robot is equivalent to the value of H_i. Thus, the minimum radius of the circles is the value of the objective function $\mathcal{H}$. The time response of H is shown in Figure 5.2. Although it is shown that $\mathcal{H}$ is non-decreasing for a sufficiently small sampling interval, H is not rigorously non-decreasing in the simulation results of T_s = 0.1 and 0.01 s.

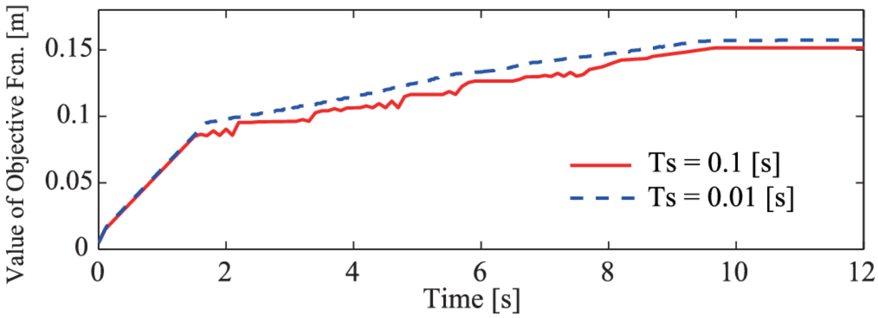

Figure 5.2: Time response of $\mathcal{H}$ in the simulation of Section 5.1.

The solid and the dashed lines in Figure 5.2 show the time responses of $\mathcal{H}$ for $T_s = 0.1$ and 0.01, respectively, from the same initial positions in Figure 5.1(left). The decrease in $\mathcal{H}$ is negligible for $T_s = 0.01$, where the largest decrease for sampling intervals is about 5.0×10^{-4}. On the other hand, the graph in is more oscillatory for $T_s = 0.1$, where the largest decrease is about 5.0×10^{-3}. Nevertheless, H successfully grows to a value similar to that in the case of $T_s = 0.01$.

5.2) VARIABLE RECTANGULAR FORMATION

We consider a variable rectangular formation with $\dot{\sigma}_1 > 0$ and $\dot{\sigma}_2 < 0$. The value of σ is changed with $\dot{\sigma}_1 = K_1 (\sigma_{d1} - \sigma_1)$ and $\dot{\sigma}_2 = K_2 (\sigma_{d2} - \sigma_2)$, where $K_1 = K_2 = 0.05$. The target value of σ is $\sigma_d = [8, 2]^T$, while the initial value is $\sigma(0) = [4, 4]^T$. To test the influence of the change in σ on $\mathcal{H}$, the initial positions of 50 robots are generated by applying the control law for the initial rectangle until $\mathcal{H}$ converges from randomly selected initial positions. As a result, $\mathcal{H}$ is not changed from the initial value, unless σ changes. From such simulations for 100 different sets of initial positions, we focus on the worst case, in the sense that the largest decrease from the initial value of $\mathcal{H}$ is observed for $\hat{K} = 0.1$.

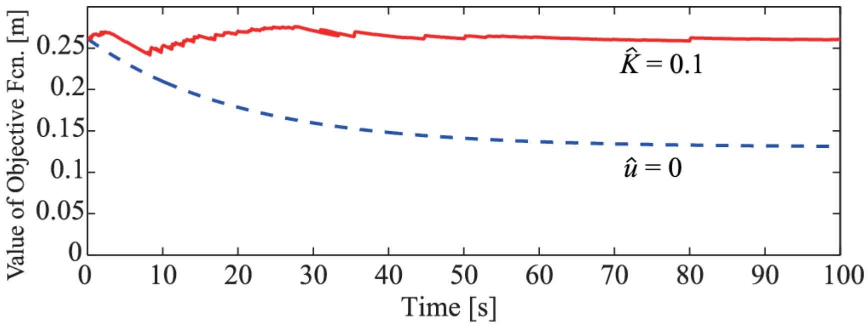

Figure 5.3: Time response of $\mathcal{H}$ for the simulation in Section 5.2.

The time response of $\mathcal{H}$ in this case is shown in the solid line in Figure 5.3. The largest decrease from the initial value is about 1.9 × 10−2 m. On the other hand, as shown in the dashed line, the value of $\mathcal{H}$ is monotonically decreased to about half of the initial value if the control input for the subtask is not applied.

5.3) BENDING FORMATION

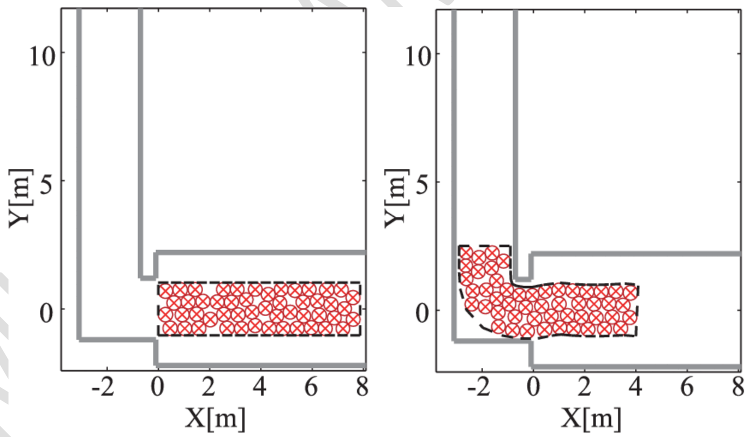

Figure 5.4: Formation in a narrow corner at t = 0 (left) and t = 60 (right).

For the rectangular formation of $\sigma = [8, 2]^T$ obtained in Section 5.3 to make a turn in the narrow corner as shown in Figure 5.4, $\dot{p}_0$ and $\dot{\theta}_0$ are given as follows:

$$\dot{p}_0(t) = \begin{cases} \begin{bmatrix} -v_1 \\ 0 \end{bmatrix}, & \text{if } 0 \le t < t_1 \\ \begin{bmatrix} -v_2 \sin \omega t \\ -v_2 \cos \omega t \end{bmatrix}, & \text{if } t_1 \le t < t_1 + t_2 \\ \begin{bmatrix} 0 \\ v_1 \end{bmatrix}, & \text{if } t_1 + t_2 \le t \end{cases}$$

$$\dot{\theta}_0(t) = \begin{cases} 0, & \text{if } 0 \le t < t_1 \\ -\omega, & \text{if } t_1 \le t < t_1 + t_2 \\ 0, & \text{if } t_1 + t_2 \le t \end{cases}$$

where $v_1 = 0.1$, $v_2 = 0.05$, $\omega = v_2/1.4$, $t_1 = 5$and $t_2 = \pi/2\omega$. Thus, p_0 and θ_0 can be obtained from the initial values $p_0(0) = [0, 0]^T$ and $\theta_0(0) = 0$. Then, $\dot{\theta}_k$ (k=1,…,m−1) is determined, where $m = \sigma_1/L$, and L = 0.5 m. We use 100 sets of initial positions of the robots, which are obtained as the final positions in the 100 cases in Section 5.2, and focus on the worst case, in the sense that the largest decrease from the initial value of H is observed for $\hat{K} = 0.1$.

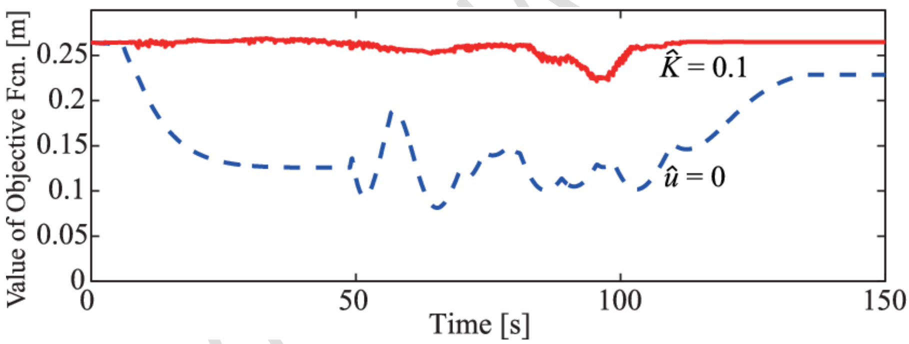

Figure 5.5: Time response of $\mathcal{H}$ for the simulation in Section 5.3.

The positions of the robots at t = 0 and t = 60 in this case are shown by “×” in Figure 5.4. The time response of $\mathcal{H}$ is shown by the solid line in Figure 5.5. The largest decrease from the initial value is about 2.8×10^{-2} m. On the other hand, if the control input us for the subtask is not applied, the value of $\mathcal{H}$ is decreased much more significantly, as shown in the dashed line, since the distances between robots become small inside the corner.

CHAPTER 6:

EXPERIMENTS

The proposed formation control method is applied to a group of eight robots. For each robot, we use a mobile robot platform “beego” (Techno Craft), which is a two-wheeled skid-steer robot with one caster wheel, as shown in Figure 6.1(left).

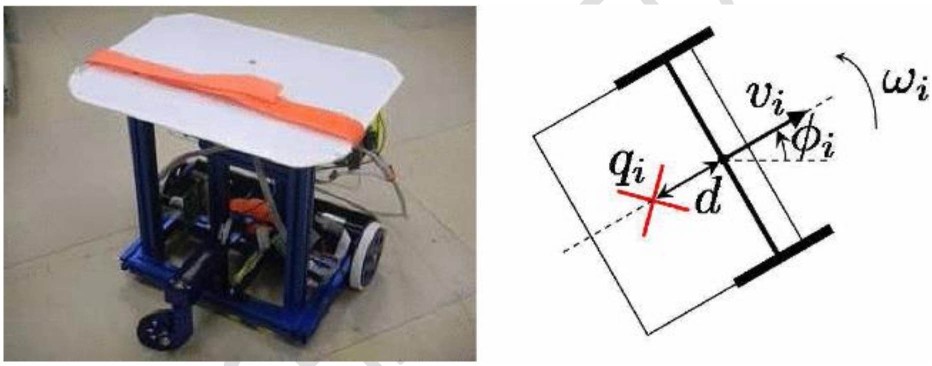

Figure 6.1: Photograph (left) and schematic (right) of a robot used in the experiments.

The model of this robot is described as follows:

$$\begin{bmatrix} \dot{X}_i \\ \dot{Y}_i \\ \dot{\phi}_i \end{bmatrix} = \begin{bmatrix} \cos\phi_i & 0 \\ \sin\phi_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}, \quad i = 1, \ldots, 8$$

where (X_i, Y_i) denotes the position of the center of the axle, and φ_i is the orientation of the robot i. The control inputs of this system are the translational and angular velocities (v_i, ω_i). To obtain the model as for this system, we define q_i as the point with an offset d = 0.1 m from the center of the axle, as shown in Figure 6.1(right).

Since we obtain

$$\dot{q}_i = B_i \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = u_i, \quad B_i := \begin{bmatrix} \cos\phi_i & d\sin\phi_i \\ \sin\phi_i & -d\cos\phi_i \end{bmatrix},$$

the input u_i, which is determined from the control algorithm, can be transformed to (v_i, ω_i) and applied to the real robot. The position and orientation of the vehicle are measured by dead reckoning.

6.1) VARIABLE RECTANGULAR FORMATION

In the same way as in Section 5.2, we consider a variable rectangular formation. The target value of σ is $\sigma_d = [3.2, 1.0]^T$,while the initial value is $\sigma(0) = [1.6, 2.0]^T$. The values of the feedback gains K_1 , K_2 , and $\hat{K}$ are the same as in Section 5.2. To choose the initial positions of the robots, we perform a simulation in which the control law is applied to the fixed rectangleof $\sigma = [1.6, 2.0]^T$ until $\mathcal{H}$ converges from a set of randomlyselected initial positions of the robots. Based on the simulation result, we choose the initial positions as follows:

$$q_1 = \begin{bmatrix} 0.3 \\ 0.7 \end{bmatrix}, \quad q_2 = \begin{bmatrix} 0.3 \\ -0.15 \end{bmatrix}, \quad q_3 = \begin{bmatrix} 0.4 \\ -0.7 \end{bmatrix}, \quad q_4 = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}$$
$$q_5 = \begin{bmatrix} 0.9 \\ -0.3 \end{bmatrix}, \quad q_6 = \begin{bmatrix} 1.2 \\ 0.7 \end{bmatrix}, \quad q_7 = \begin{bmatrix} 1.3 \\ 0.1 \end{bmatrix}, \quad q_8 = \begin{bmatrix} 1.3 \\ -0.7 \end{bmatrix}.$$

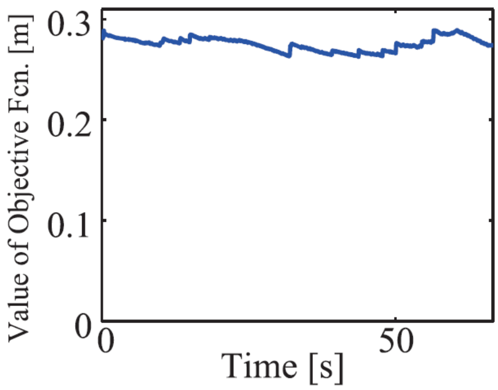

Figure 6.2: Time response of $\mathcal{H}$ for the experiments in Section 6.1.

Figure 6.2 shows that the decrease in the value of $\mathcal{H}$ during the formation change is kept small. The largest decrease from the initial value is about 1.7×10^{-2} m.

6.2) BENDING FORMATION

From the initial rectangular formation with $\sigma_d = [3.2, 1.0]^T$,a group of robots makes a turn by giving $\dot{p}_0$ and $\dot{\theta}_0$, where $v_1 = 0.05$, $v_2 = 0.025$, $\omega = v_2/0.96$, $t_1 = 3.9$, and $t_2 = \pi/2\omega$.The initial values of p_0 and θ_0 are $[0,0]^T$ and 0, respectively. To choose the initial positions of the robots, we perform a simulation in which the control law is applied to the fixed rectangle of $\sigma = [3.2, 1.0]^T$ until $\mathcal{H}$ converges from a set of randomly selected initial positions of the robots. Based on the simulation result, we choose the initial positions as follows:

$$q_1 = \begin{bmatrix} 0.30 \\ -0.21 \end{bmatrix}, \quad q_2 = \begin{bmatrix} 0.70 \\ 0.22 \end{bmatrix}, \quad q_3 = \begin{bmatrix} 1.06 \\ -0.23 \end{bmatrix}, \quad q_4 = \begin{bmatrix} 1.41 \\ 0.23 \end{bmatrix}$$

$$q_5 = \begin{bmatrix} 1.76 \\ -0.23 \end{bmatrix}, \quad q_6 = \begin{bmatrix} 2.11 \\ 0.23 \end{bmatrix}, \quad q_7 = \begin{bmatrix} 2.47 \\ -0.23 \end{bmatrix}, \quad q_8 = \begin{bmatrix} 2.86 \\ 0.21 \end{bmatrix}.$$

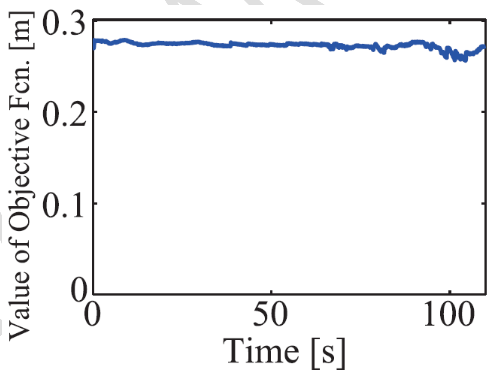

Figure 6.3: Time response of $\mathcal{H}$ for the experiments in Section 6.2.

Figure 6.3 shows that the decrease in the value of $\mathcal{H}$ during the formation change is kept small. The largest decrease from the initial value is about 1.4×10^{-2} m.

CHAPTER 7:
CONCLUSIONS

In the present technological scenario of cooperative robotics research, a new method of controlling a group of mobile robots based on formation abstraction is introduced. In rectangular formations, a deformable polygonal formation is used to go through narrow spaces without colliding with obstacles. Furthermore, the robots continuously try to optimize their positions to decrease the risk of collisions by integrating a decentralized locational optimization algorithm into formation control. We have shown that the objective function for the locational optimization does not decrease for fixed non-convex polygonal formation shapes if the zero-order hold control is applied for a sufficiently short sampling period. Then for the variable rectangular formation, we have also shown that the locational optimization algorithm makes the value of the objective function greater than the value in the case without the locational optimization. The effectiveness of the proposed method has been demonstrated in both simulations and real robot experiments.

However, more studies are necessary in the future for an actual implementation. Since it is inevitable that the objective function is not necessarily non-decreasing in variable formation cases, robots need to have some margins in distances from other robots and from the boundary of the polygonal region in order to achieve two goals, i.e., maintaining robots in the region and avoiding collisions, which often conflict with each other. To this end, an important problem is how to determine the area of the polygonal region. It is also important to determine the speed of deformation appropriately, since the numbers of margins required depend on the speed of deformation. Emergency mechanisms, which change the priorities of two goals in the case where one of the two goals has to be given up to achieve, also need to be studied. In addition, although we assume that parameters such as L and σ_d are given in this paper, the algorithm to decide them on the basis of information on the environments should be studied in the future.

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